Dynamical trapping of light in modulated waveguide lattices

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A discrete analogue of the dynamical (Kapitza) trapping effect, known for classical and quantum particles in rapidly oscillating potentials, is proposed for light waves in modulated graded-index waveguide lattices. As in the non-modulated waveguide lattice a graded-index potential can confine light at either normal or Bragg angle incidence, periodic modulation of the potential in the longitudinal direction enables to trap optical beams at both normal and Bragg incidence angles.

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Light propagation in photonic lattices has attracted a great interest over the past few years, with the observation of a host of new phenomena with no counterpart in continuous media [1, 2]. Flexible control of light transport and localization in such devices can be realized by breaking the translation invariance of the lattice along the propagation direction by either periodic axis bending or out-of-phase modulation of refractive index of adjacent guides. Examples of light control in modulated lattices include dynamic localization [3–6], tunneling inhibition [7–9], multiband refraction control [10], polychromatic diffraction management [11–13], and defect-free surface waves [14, 15]. Most of such light control techniques bear interesting analogies with coherent control of driven quantum systems, such as electronic or matter wave transport in driven lattices [16].

In this Letter a mechanism of light trapping in graded-index modulated waveguide lattices is proposed, which is based on a *discrete analogue* of the Kapitza (or dynamical) stabilization effect of classical and quantum particles in rapidly oscillating potentials [17–21]. Dynamical stabilization generally refers to the possibility for a particle to be trapped by a rapidly-oscillating potential in cases where the static potential cannot. Well-known paradigms are the Kapitza stabilization of the pendulum [17, 18] and Paul traps for charged particles [19]. Here it is shown that modulated lattices can trap light beams under conditions where the non-modulated lattice cannot. Let us consider light propagation in a modulated waveguide lattice [7], which in the tight-binding approximation is governed by the discrete Schrödinger equation

$$i\frac{d\psi_n}{dz} = -\kappa(\psi_{n+1} + \psi_{n-1}) + \Phi_n(z)\psi_n \ , \qquad (1)$$

where $\psi_n(z)$ is the light field amplitude trapped in the nth waveguide, z is the longitudinal propagation coordinate, $\kappa > 0$ is the coupling constant between adjacent waveguides, and $\Phi_n(z)$ is the longitudinal modulation of the propagation constant for the nth guide. In the following, we will assume $\Phi_n(z) = f(z)V_n$, where $f(z)$ is a modulation function with spatial frequency $\omega = 2\pi/\Lambda$ and zero mean, and V_n is the graded-index (static) potential. In waveguide arrays manufactured by femtosecond laser writing, the modulation can be realized by slightly varying the writing speed for each waveguide [7]. In the absence of the longitudinal modulation, i.e. for $f = 1$, a graded-index potential V_n can be exploited to focus or trap light beams. Such a kind of focusing has been proposed, for example, to achieve deep subwavelength focusing in metal-dielectric waveguide arrays [22]. A schematic of the refractive index profile for a non-modulated graded-index array is depicted in Fig.1. To relate the propagation properties of the graded-index modulated lattice (1) with the dynamics of a quantum particle in a rapidly-oscillating potential [20, 21], it is worth observing that the solution to the coupled-mode equations (1) can be written as $\psi_n(z) = \psi(x = n, z)$, where the continuous function $\psi(x, z)$ satisfies the Schrödinger equation $i(\partial\psi/\partial z) = \mathcal{H}\psi$ with Hamiltonian [23]

$$\mathcal{H} = -2\kappa\cos(p_x) + f(z)V(x) \qquad (2)$$

where $p_x = -i\partial/\partial_x$ and $V(x = n) = V_n$. For a potential V that varies slowly over one lattice period and for a wave packet with a narrow angular spectrum distribution, the semiclassical (ray optics) equations for the mean values $\langle x\rangle$ of wave packet position (in units of the array period) and $\langle p_x\rangle$ of refraction angle, as given by the Ehrenfest theorem, read [23]

$$\frac{d\langle x\rangle}{dz} \simeq 2\kappa\sin(\langle p_x\rangle) \ , \ \frac{d\langle p_x\rangle}{dz} \simeq -f(z)\left(\frac{\partial V}{\partial x}\right)(\langle x\rangle, z). \qquad (3)$$

Let us assume that the static potential $V(x)$ is a bell-shaped potential, for example, described by a parabolic or a Gaussian function, with a maximum at $x = 0$, and let us first consider the non-modulated array ($f = 1$). Then, according to Eqs.(3) there are two fixed points, ($\langle x\rangle = 0, \langle p_x\rangle = 0$) and ($\langle x\rangle = 0, \langle p_x\rangle = \pi$), the former being unstable and the latter being stable [24]. This means that a broad beam launched into the array at nearly normal incidence will not be trapped by the the static potential V_n, which acts as a defocusing lens for the discretized beam. Conversely, an injected broad beam tilted near the Bragg angle (corresponding to $\langle p_x\rangle \simeq \pi$) will be confined by the potential V_n, which acts as a focusing lens. This is shown, as an example, in Figs.1(a)

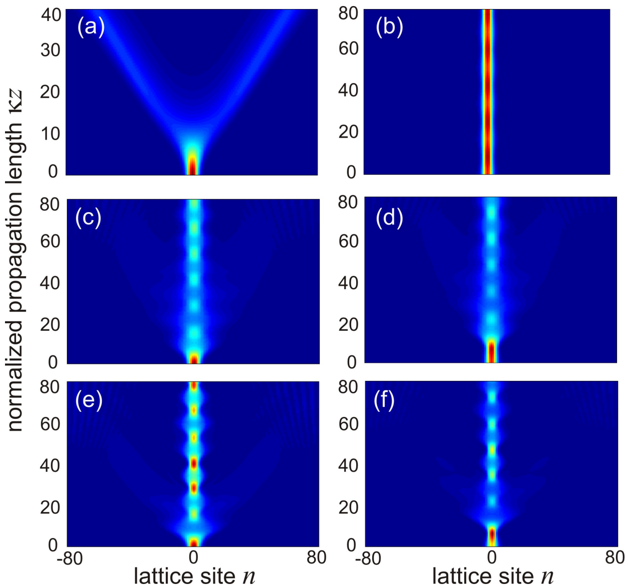

Fig. 1. (Color online) Beam propagation (snapshot of $|\psi_n(z)|^2$) in a modulated lattice (square-wave modulation, $\omega/\kappa = 0.5$) with a Gaussian-shaped graded index potential (peak amplitude V_0) for normal and Bragg incidence angles and for increasing values of normalized potential peak amplitude: $V_0/\kappa = 0$ in (a) and (b) (non-modulated lattice); $V_0/\kappa = 3$ in (c) and (d); $V_0/\kappa = 5$ in (e) and (f). The right panels schematically show the index profile of the non-modulated (upper plot) and modulated (lower plot) graded-index waveguide arrays.

and (b). If the sign of the potential V_n is reversed, the stability of the two fixed points is interchanged. Such a behavior is related to the well-known reversal of diffraction sign for discretized light at normal or Bragg beam incidence [1, 25]. In fact, in the former case (normal incidence) the Hamiltonian $\mathcal{H}$ is approximated as $\mathcal{H} \simeq -\kappa\partial_x^2 - 2\kappa + V(x)$, whereas in the latter case (Bragg angle incidence) one has $\mathcal{H} \simeq \kappa\partial_x^2 + 2\kappa + V(x)$. Let us now consider the modulated lattice, and show that, similarly to the dynamical trapping of classical or quantum particles in rapidly-oscillating potentials [17, 18, 20, 21], dynamical beam trapping in the lattice can be realized for *both* normal and Bragg beam incidence. Indeed, propagation of a broad beam, at either normal or Bragg incidence angles, is governed by the Schrödinger-type equation with an oscillating potential, namely

$$i\frac{\partial\psi}{\partial z} \simeq \mp\kappa\frac{\partial^2\psi}{\partial x^2} \mp 2\kappa\psi + f(z)V(x)\psi \tag{4}$$

where the upper (lower) sign applies to normal (Bragg) incidence. For a rapidly oscillating potential, after setting $\psi(x,z) = \phi(x,z)\exp\left[-iV(x)\left(\int_0^z d\xi f(\xi) - \overline{\int_0^z d\xi f(\xi)}\right)\right]$ and applying standard averaging methods [20, 21], at leading order the evolution of the slowly-varying envelope $\phi(x,z)$ is described by a Schrödinger-type equation with an effective static potential $V_e(x)$, namely $i\partial_z\phi = [\mp\kappa\partial_x^2\phi + V_e(x)]\phi$, where

$$V_e(x) = \mp 2\kappa \pm \kappa\left(\frac{\partial V}{\partial x}\right)^2 \overline{\left(\int_0^z d\xi f(\xi) - \overline{\int_0^z d\xi f(\xi)}\right)^2} \tag{5}$$

and the overbar denotes a spatial average over the oscillation cycle. For a bell-shaped potential V and considering normal beam incidence, the effective potential $V_e(x)$ comprises two potential barriers, which can support metastable (resonance) states (see, for instance, [21]). Hence, though the static potential (i.e. for $f = 1$) can not trap light beams at normal incidence near $x = 0$, in the modulated lattice this is possible owing to the dynamical (Kapitza) stabilization effect. For beam incidence at the Bragg angle, the sign of both diffraction and effective potential are reversed, and thus trapping is possible as well. It should be noted that the strength of the confining part of the effective potential is usually very small (it scales as $\sim 1/\omega^2$ [20, 21]), and the observation of such a dynamical stabilization effect for broad light beams in waveguide lattices might require extremely long propagation distances, which are not accessible with current waveguide array set-ups. Remarkably, we found that the dynamical trapping effect persists even for relatively small spatial modulation frequencies ω, of the order or smaller than the coupling constant κ, making dynamical trapping observable with current waveguide array set-ups. As an example, Fig.1 shows dynamical trapping of a broad Gaussian beam, at both normal and Bragg incidence angles, for a Gaussian-shaped potential $V_n = V_0\exp[-(n/w)^2]$ and for a square-wave modulation function [$f(z) = 1$ in one semicycle, and $f(z) = -1$ in the other semicycle], as obtained by direct numerical simulations of Eqs.(1) for $\omega/\kappa = 0.5$, $w = 18$ and for increasing values of V_0/κ. Initial condition is $c_n(0) = \exp(-n^2/25)$ in (a), (c) and (e) (normal incidence), and $c_n(0) = (-1)^n\exp(-n^2/25)$ in (b),(d) and (f) (Bragg incidence angle). For a typical coupling constant of $\kappa \simeq 0.5$ mm^{-1} [7], a propagation length of 40 in Fig.1 corresponds to a physical length of $\simeq 8$ cm. Dynamical stabilization, observed in Figs.1(c-f), is related to the existence of metastable (resonance) states of the modulated lattice, which can be revealed by a direct computation of the quasienergy spectrum and Floquet eigenstates of the coupled-mode equations (1) with periodic coefficients. The quasi-energies (Floquet exponents) μ and corresponding Floquet eigenstates $\psi_n^{(\mu)}(z)$ are defined as the solutions to Eqs.(1) of the form $\psi_n^{(\mu)}(z) = u_n^{(\mu)}(z)\exp(-i\mu z)$ with $u_n^{(\mu)}(z+\Lambda) = u_n^{(\mu)}(z)$ and $-\omega/2 \leq \mu < \omega/2$. In a truncated lattice, a metastable (resonance) state to Eqs.(1) corresponds to a Floquet state which is strongly localized near $n = 0$. For a modulation function $f(z)$ satisfying the symmetry condition $f(-z) = f(z)$, as the one used in the simulations of Fig.1, it can be readily shown that, if $\psi_n^{(\mu)}(z) = u_n^{(\mu)}(z)\exp(-i\mu z)$ is a Floquet eigenstate with quasienergy μ, then $\psi_n^{(-\mu)}(z) = (-1)^n u_n^{(\mu)}(-z)\exp(i\mu z)$

is also a Floquet eigenstate with quasienergy $-\mu$. Therefore, resonance states appear in pairs. For the modulated lattice with parameter values used in the simulations of Fig.1, a direct computation of the Floquet exponents (assuming a truncated waveguide array comprising $2N+1=161$ waveguides, from $n=-N$ to $n=N$) shows the existence of one pair of metastable states with quasienergies $\pm 0.0284\kappa$ at $V_0/\kappa = 3$, and of two pairs of metastable states (quasienergies $\simeq \pm 0.0533\kappa$ and $\simeq \pm 0.1424\kappa$) at $V_0/\kappa = 5$. As a general rule, for a fixed modulation frequency the number of resonance states increases as the amplitude V_0 of the static potential is increased, which is in agreement with the prediction based on the cycle-averaged model: in that case the number of resonances sustained by the double-barrier effective static potential (5) increases as V_0 is increased. The profiles of the two resonance states at $V_0/\kappa = 3$, corresponding to the dynamical trapping of Figs.1(c) and (d), are depicted in Fig.2. In this case, the metastable state $\psi_n^{(-\mu)}$ with negative quasienergy is mainly excited by a normal incidence input beam, whereas the metastable state $\psi_n^{(\mu)}$ with positive quasienergy is the mainly excited state when a beam incident at the Bragg angle is considered.

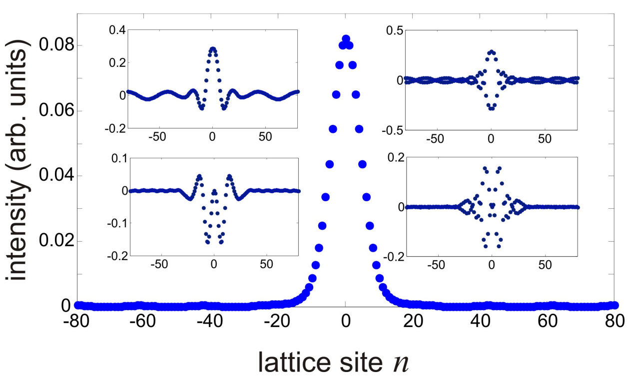

Fig. 2. (Color online) Numerically-computed intensity distribution $|\psi_n^{(\pm\mu)}(z)|^2$ of the two metastable Floquet states, at the plane $z=0$, for the modulated waveguide lattice of Fig.1 with $V_0/\kappa = 3$. The insets in the figure show the detailed behavior of the real (upper panels) and imaginary (lower panels) parts of the two Floquet states with negative (left plots) and positive (right plots) quasienergy.

In conclusion, a discrete analogue of dynamical trapping of classical or quantum particles in rapidly oscillating potentials has been proposed for light waves in photonic lattices. Such an analogy could be exploited to trap light beams at different incidence angles, thus increasing the coupling efficiency from a broad angular spectrum light source. However, as compared to ordinary focusing and trapping in non-modulated graded-index lattices, dynamical trapping does not sustain truly guided modes, and requires a more complex refractive index management. Possible applications could be envisaged, for example, in light trapping at the sub-wavelength regime [22].

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References

1. F. Lederer, G. I. Stegeman, D. N. Christodoulides, G. Assanto, M. Segev, and Y. Silberberg, Phys. Rep. **463**, 1 (2008).
2. Y. V. Kartashov, V. A. Vysloukh, and L. Torner, Prog. Opt. **52**, 63 (2009).
3. S. Longhi, Opt. Lett. **30**, 2137 (2005).
4. S. Longhi, M. Marangoni, M. Lobino, R. Ramponi, P. Laporta, E. Cianci, and V. Foglietti, Phys. Rev. Lett. **96**, 243901 (2006).
5. R. Iyer, J.S. Aitchison, J. Wan, M.M. Dignam, and C.M de Sterke, Opt. Express **15**, 3212 (2007).
6. A. Szameit, I.L. Garanovich, M. Heinrich, A.A. Sukhorukov, F. Dreisow, T. Pertsch, S. Nolte, A. Tünnermann, S. Longhi, and Y.S. Kivshar, Phys. Rev. Lett. **104**, 223903 (2010).
7. A. Szameit, Y.V. Kartashov, F. Dreisow, M. Heinrich, T. Pertsch, S. Nolte, A. Tünnermann, V.A. Vysloukh, F. Lederer, and L. Torner, Phys. Rev. Lett. **102**, 153901 (2009).
8. Y.V. Kartashov, A. Szameit, V.A. Vysloukh, and L. Torner, Opt. Lett. **34**, 2906 (2009).
9. Y.V. Kartashov and V.A. Vysloukh, Opt. Lett. **35**, 205 (2010).
10. S. Longhi, Opt. Lett. **31**, 1857 (2006).
11. I.L. Garanovich, A.A. Sukhorukov, and Y.S. Kivshar, Phys. Rev. E **74** 066609 (2006).
12. A. Szameit, I.L. Garanovich, M. Heinrich, A.A. Sukhorukov, F. Dreisow, T. Pertsch, S. Nolte, A. Tünnermann, and Y.S. Kivshar, Nat. Phys. **5**, 271 (2009).
13. X.Y. Qi, I.L. Garanovich, A.A. Sukhorukov, W. Krolikowski, A. Mitchell, G.Q. Zhang, D.N. Neshev, and Y.S. Kivshar, Opt. Lett. **35**, 1371 (2010).
14. I.L. Garanovich, A.A. Sukhorukov, and Y.S. Kivshar, Phys. Rev. Lett. **100**, 203904 (2008).
15. A. Szameit, I.L. Garanovich, M. Heinrich, A.A. Sukhorukov, F. Dreisow, T. Pertsch, S. Nolte, A. Tünnermann, and Y.S. Kivshar, Phys. Rev. Lett. **101**, 203902 (2008).
16. S. Longhi, Laser and Photon. Rev. **3**, 243 (2009).
17. P.L. Kapitza, Sov. Phys. JETP **21**, 588 (1951).
18. L.D. Landau and E.M. Lifshitz, *Mechanics* (Pergamon, Oxford, 1960), pp. 93-95.
19. W. Paul, Rev. Mov. Phys. **62**, 531 (1990).
20. R.J. Cook, D.G. Shankland, and A.L. Wells, Phys. Rev. A **31**, 564 (1985).
21. I. Gilary, N. Moiseyev, S. Rahav, and S. Fishman, J. Phys. A **36**, L409 (2003).
22. L. Verslegers, P.B. Catrysse, Z. Yu, and S. Fan, Phys. Rev. Lett. **103**, 033902 (2009).
23. S. Longhi, Phys. Rev. B **76**, 195119 (2007).
24. For a parabolic potential, Eqs.(3) reduce to the pendulum equations; the two fixed points correspond to the unstable and stable stationary points of the pendulum.
25. T. Pertsch, T. Zentgraf, U. Peschel, A. Bräuer, and F. Lederer, Phys. Rev. Lett. **88**, 093901 (2002).